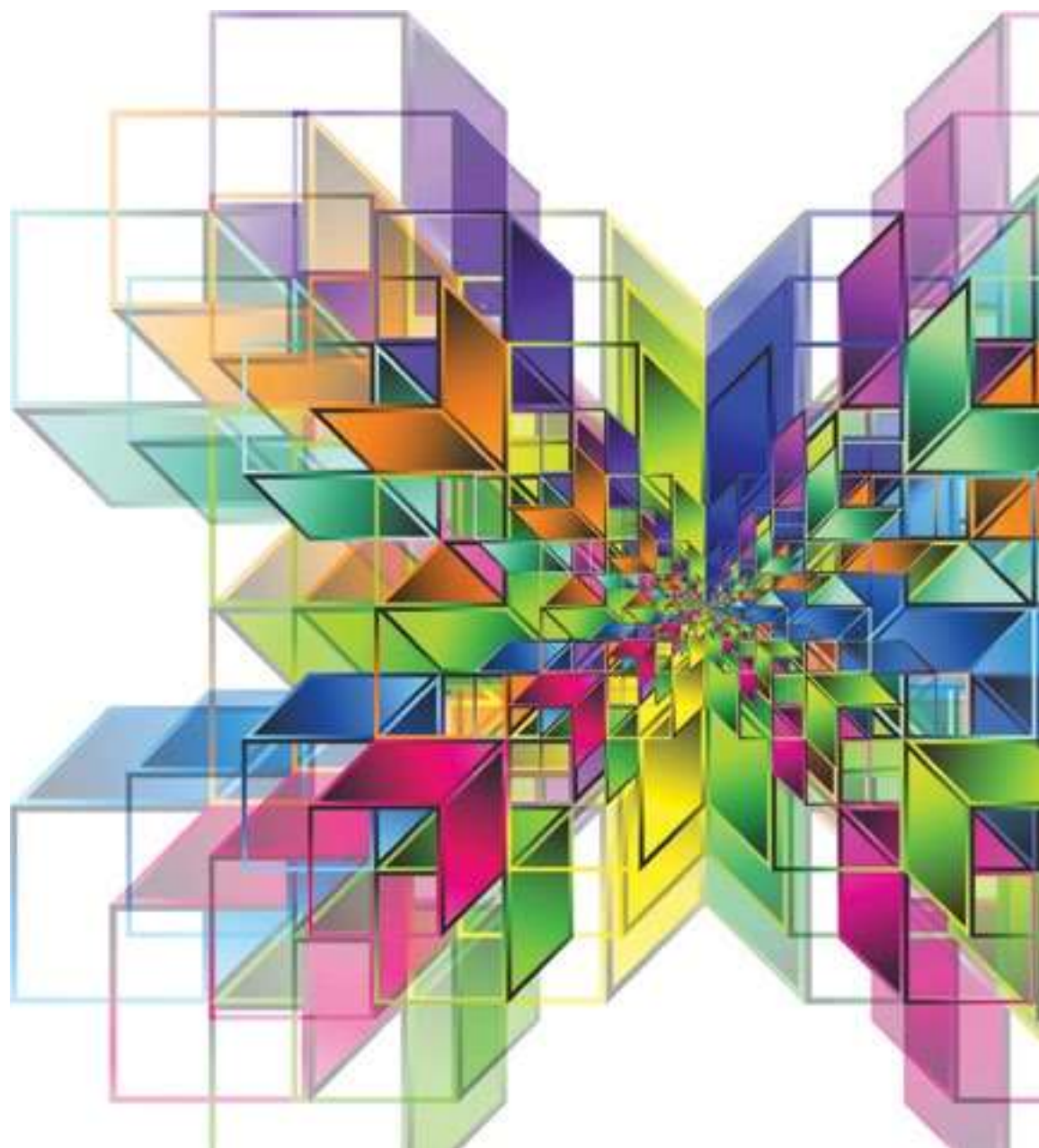

GCSE MATHEMATICS

(8300)

Specification

For teaching from September 2015 onwards
For exams in May/June 2017 onwards

Version 1.0 12 September 2014



All content can be assessed on Higher tier question papers.

Notes are added to exemplify some of the specification references.

In addition to this subject content, students should be able to recall, select and apply mathematical formulae. See the [Appendix](#) for a list of the DfE prescribed formulae.

3.1 Number

3.1.1 Structure and calculation

N1

Basic foundation content	Additional foundation content	Higher content only
order positive and negative integers, decimals and fractions use the symbols =, ≠, <, >, ≤, ≥		

Notes: including use of a number line. See also [A22](#)

N2

Basic foundation content	Additional foundation content	Higher content only
apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative understand and use place value (eg when working with very large or very small numbers, and when calculating with decimals)		

Notes: including questions set in context.

Knowledge and understanding of terms used in household finance, for example profit, loss, cost price, selling price, debit, credit, balance, income tax, VAT and interest rate. See also [R9](#)

N3

Basic foundation content	Additional foundation content	Higher content only
<p>recognise and use relationships between operations, including inverse operations (eg cancellation to simplify calculations and expressions)</p> <p>use conventional notation for priority of operations, including brackets, powers, roots and reciprocals</p>		

N4

Basic foundation content	Additional foundation content	Higher content only
<p>use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem</p>		

Notes: prime factor decomposition including product of prime factors written in index form.

N5

Basic foundation content	Additional foundation content	Higher content only
<p>apply systematic listing strategies</p>		<p>including use of the product rule for counting</p>

Notes: including using lists, tables and diagrams.

N6

Basic foundation content	Additional foundation content	Higher content only
<p>use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5</p>		<p>estimate powers and roots of any given positive number</p>

Notes: including square numbers up to 15×15

Students should know that $1000 = 10^3$ and 1 million = 10^6

N7

Basic foundation content	Additional foundation content	Higher content only
	calculate with roots, and with integer indices	calculate with fractional indices

N8

Basic foundation content	Additional foundation content	Higher content only
calculate exactly with fractions	calculate exactly with multiples of π	calculate exactly with surds simplify surd expressions involving squares (eg $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$) and rationalise denominators

Notes: see also [G17](#) and [G18](#)

N9

Basic foundation content	Additional foundation content	Higher content only
calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer		

Notes: with and without a calculator.

Interpret calculator displays.

3.1.2 Fractions, decimals and percentages

N10

Basic foundation content	Additional foundation content	Higher content only
work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)		change recurring decimals into their corresponding fractions and vice versa

Notes: including ordering.

N11

Basic foundation content	Additional foundation content	Higher content only
identify and work with fractions in ratio problems		

Notes: See also [R8](#)

N12

Basic foundation content	Additional foundation content	Higher content only
interpret fractions and percentages as operators		

Notes: including interpreting percentage problems using a multiplier. See also [R9](#)

3.1.3 Measures and accuracy**N13**

Basic foundation content	Additional foundation content	Higher content only
use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate		

Notes: know and use metric conversion factors for length, area, volume and capacity.

Imperial/metric conversions will be given in the question.

N14

Basic foundation content	Additional foundation content	Higher content only
estimate answers check calculations using approximation and estimation, including answers obtained using technology		

Notes: including evaluation of results obtained. See also [N15](#)

N15

Basic foundation content	Additional foundation content	Higher content only
round numbers and measures to an appropriate degree of accuracy (eg to a specified number of decimal places or significant figures)	use inequality notation to specify simple error intervals due to truncation or rounding	

Notes: including appropriate rounding for questions set in context.

Students should know not to round values during intermediate steps of a calculation. See also [N14](#)

N16

Basic foundation content	Additional foundation content	Higher content only
	apply and interpret limits of accuracy	including upper and lower bounds

3.2 Algebra

3.2.1 Notation, vocabulary and manipulation

A1

Basic foundation content	Additional foundation content	Higher content only
<ul style="list-style-type: none">• use and interpret algebraic notation, including:• ab in place of $a \times b$• $3y$ in place of $y + y + y$ and $3 \times y$• a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$• $\frac{a}{b}$ in place of $a \div b$• coefficients written as fractions rather than as decimals• brackets		

Notes: it is expected that answers will be given in their simplest form without an explicit instruction to do so.

A2

Basic foundation content	Additional foundation content	Higher content only
substitute numerical values into formulae and expressions, including scientific formulae		

Notes: unfamiliar formulae will be given in the question.

See the Appendix for a full list of the prescribed formulae. See also [A5](#)

A3

Basic foundation content	Additional foundation content	Higher content only
understand and use the concepts and vocabulary of expressions, equations, formulae, inequalities, terms and factors	to include identities	

Notes: this will be implicitly and explicitly assessed.

A4

Basic foundation content	Additional foundation content	Higher content only
simplify and manipulate algebraic expressions by:	simplify and manipulate algebraic expressions (including those involving surds) by:	simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions) by:
<ul style="list-style-type: none"> collecting like terms multiplying a single term over a bracket taking out common factors simplifying expressions involving sums, products and powers, including the laws of indices 		
	<ul style="list-style-type: none"> expanding products of two binomials factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares 	<ul style="list-style-type: none"> expanding products of two or more binomials factorising quadratic expressions of the form $ax^2 + bx + c$

A5

Basic foundation content	Additional foundation content	Higher content only
understand and use standard mathematical formulae rearrange formulae to change the subject		

Notes: including use of formulae from other subjects in words and using symbols.

See the Appendix for a full list of the prescribed formulae. See also [A2](#)

A6

Basic foundation content	Additional foundation content	Higher content only
	know the difference between an equation and an identity	
	argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments	to include proofs

A7

Basic foundation content	Additional foundation content	Higher content only
where appropriate, interpret simple expressions as functions with inputs and outputs		interpret the reverse process as the 'inverse function' interpret the succession of two functions as a 'composite function'

Notes: understanding and use of $f(x)$, $fg(x)$ and $f^{-1}(x)$ notation is expected at Higher tier.

3.2.2 Graphs

A8

Basic foundation content	Additional foundation content	Higher content only
work with coordinates in all four quadrants		

A9

Basic foundation content	Additional foundation content	Higher content only
plot graphs of equations that correspond to straight-line graphs in the coordinate plane	use the form $y = mx + c$ to identify parallel lines find the equation of the line through two given points, or through one point with a given gradient	use the form $y = mx + c$ to identify perpendicular lines

A10

Basic foundation content	Additional foundation content	Higher content only
identify and interpret gradients and intercepts of linear functions graphically and algebraically		

A11

Basic foundation content	Additional foundation content	Higher content only
	identify and interpret roots, intercepts and turning points of quadratic functions graphically	
	deduce roots algebraically	deduce turning points by completing the square

Notes: including the symmetrical property of a quadratic. See also [A18](#)

A12

Basic foundation content	Additional foundation content	Higher content only
recognise, sketch and interpret graphs of linear functions and quadratic functions	including simple cubic functions and the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$	including exponential functions $y = k^x$ for positive values of k , and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size

Notes: see also [G21](#)

A13

Basic foundation content	Additional foundation content	Higher content only
		sketch translations and reflections of a given function

A14

Basic foundation content	Additional foundation content	Higher content only
plot and interpret graphs, and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration	including reciprocal graphs	including exponential graphs

Notes: including problems requiring a graphical solution. See also [A15](#)

A15

Basic foundation content	Additional foundation content	Higher content only
		calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts

Notes: see also [A14](#), [R14](#) and [R15](#)

A16

Basic foundation content	Additional foundation content	Higher content only
		recognise and use the equation of a circle with centre at the origin find the equation of a tangent to a circle at a given point

3.2.3 Solving equations and inequalities

A17

Basic foundation content	Additional foundation content	Higher content only
solve linear equations in one unknown algebraically find approximate solutions using a graph	including those with the unknown on both sides of the equation	

Notes: including use of brackets.

A18

Basic foundation content	Additional foundation content	Higher content only
	solve quadratic equations algebraically by factorising find approximate solutions using a graph	including those that require rearrangement including completing the square and by using the quadratic formula

Notes: see also [A11](#)

A19

Basic foundation content	Additional foundation content	Higher content only
	solve two simultaneous equations in two variables (linear/linear) algebraically find approximate solutions using a graph	including linear/quadratic

A20

Basic foundation content	Additional foundation content	Higher content only
		find approximate solutions to equations numerically using iteration

Notes: including the use of suffix notation in recursive formulae.

A21

Basic foundation content	Additional foundation content	Higher content only
	translate simple situations or procedures into algebraic expressions or formulae derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution	

Notes: including the solution of geometrical problems and problems set in context.

A22

Basic foundation content	Additional foundation content	Higher content only
	solve linear inequalities in one variable	solve linear inequalities in one or two variable(s), and quadratic inequalities in one variable
	represent the solution set on a number line	represent the solution set on a number line, using set notation and on a graph

Notes: students should know the conventions of an open circle on a number line for a strict inequality and a closed circle for an included boundary. See also [N1](#)

In graphical work the convention of a dashed line for strict inequalities and a solid line for an included inequality will be required.

3.2.4 Sequences

A23

Basic foundation content	Additional foundation content	Higher content only
generate terms of a sequence from either a term-to-term or a position-to-term rule		

Notes: including from patterns and diagrams.

A24

Basic foundation content	Additional foundation content	Higher content only
recognise and use sequences of triangular, square and cube numbers and simple arithmetic progressions	including Fibonacci-type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer and r is a rational number > 0)	including other sequences including where r is a surd

Notes: other recursive sequences will be defined in the question.

A25

Basic foundation content	Additional foundation content	Higher content only
deduce expressions to calculate the n th term of linear sequences		including quadratic sequences

3.3 Ratio, proportion and rates of change

R1

Basic foundation content	Additional foundation content	Higher content only
change freely between related standard units (eg time, length, area, volume/capacity, mass) and compound units (eg speed, rates of pay, prices) in numerical contexts	compound units (eg density, pressure) in numerical and algebraic contexts	

R2

Basic foundation content	Additional foundation content	Higher content only
use scale factors, scale diagrams and maps		

Notes: including geometrical problems.

R3

Basic foundation content	Additional foundation content	Higher content only
express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1		

R4

Basic foundation content	Additional foundation content	Higher content only
use ratio notation, including reduction to simplest form		

R5

Basic foundation content	Additional foundation content	Higher content only
divide a given quantity into two parts in a given part : part or part : whole ratio express the division of a quantity into two parts as a ratio apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)		

Notes: including better value or best-buy problems.

R6

Basic foundation content	Additional foundation content	Higher content only
express a multiplicative relationship between two quantities as a ratio or a fraction		

R7

Basic foundation content	Additional foundation content	Higher content only
understand and use proportion as equality of ratios		

R8

Basic foundation content	Additional foundation content	Higher content only
relate ratios to fractions and to linear functions		

Notes: see also [N11](#), [R14](#)

R9

Basic foundation content	Additional foundation content	Higher content only
<p>define percentage as ‘number of parts per hundred’</p> <p>interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively</p> <p>express one quantity as a percentage of another</p> <p>compare two quantities using percentages</p> <p>work with percentages greater than 100%</p> <p>solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics</p>		

Notes: see also [N2](#), [N12](#)

R10

Basic foundation content	Additional foundation content	Higher content only
<p>solve problems involving direct and inverse proportion, including graphical and algebraic representations</p>		

R11

Basic foundation content	Additional foundation content	Higher content only
<p>use compound units such as speed, rates of pay, unit pricing</p>	<p>use compound units such as density and pressure</p>	

Notes: including making comparisons.

R12

Basic foundation content	Additional foundation content	Higher content only
compare lengths, areas and volumes using ratio notation scale factors	make links to similarity (including trigonometric ratios)	

Notes: see also [G19](#), [G20](#)

R13

Basic foundation content	Additional foundation content	Higher content only
	understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$	
	interpret equations that describe direct and inverse proportion	construct and interpret equations that describe direct and inverse proportion

R14

Basic foundation content	Additional foundation content	Higher content only
	interpret the gradient of a straight-line graph as a rate of change recognise and interpret graphs that illustrate direct and inverse proportion	

Notes: see also [A15](#), [R8](#)

R15

Basic foundation content	Additional foundation content	Higher content only
		interpret the gradient at a point on a curve as the instantaneous rate of change apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts

Notes: see also [A15](#)

R16

Basic foundation content	Additional foundation content	Higher content only
	set up, solve and interpret the answers in growth and decay problems, including compound interest	and work with general iterative processes

3.4 Geometry and measures**3.4.1 Properties and constructions****G1**

Basic foundation content	Additional foundation content	Higher content only
<p>use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries</p> <p>use the standard conventions for labelling and referring to the sides and angles of triangles</p> <p>draw diagrams from written description</p>		

G2

Basic foundation content	Additional foundation content	Higher content only
	<p>use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle)</p> <p>use these to construct given figures and solve loci problems</p> <p>know that the perpendicular distance from a point to a line is the shortest distance to the line</p>	

Notes: including constructing an angle of 60° .

G3

Basic foundation content	Additional foundation content	Higher content only
<p>apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles</p> <p>understand and use alternate and corresponding angles on parallel lines</p> <p>derive and use the sum of angles in a triangle (eg to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)</p>		

Notes: colloquial terms such as Z angles are not acceptable and should not be used.

G4

Basic foundation content	Additional foundation content	Higher content only
<p>derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus</p> <p>and triangles and other plane figures using appropriate language</p>		

Notes: including knowing names and properties of isosceles, equilateral, scalene, right-angled, acute-angled, obtuse-angled triangles. Including knowing names and using the polygons: pentagon, hexagon, octagon and decagon.

G5

Basic foundation content	Additional foundation content	Higher content only
	use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)	

G6

Basic foundation content	Additional foundation content	Higher content only
	apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs	

G7

Basic foundation content	Additional foundation content	Higher content only
identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement	including fractional scale factors	including negative scale factors

G8

Basic foundation content	Additional foundation content	Higher content only
		describe the changes and invariance achieved by combinations of rotations, reflections and translations

Notes: including using column vector notation for translations. See also [G24](#)

G9

Basic foundation content	Additional foundation content	Higher content only
identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference	including: tangent, arc, sector and segment	

G10

Basic foundation content	Additional foundation content	Higher content only
		apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results

Notes: including angle subtended by an arc at the centre is equal to twice the angle subtended at any point on the circumference, angle subtended at the circumference by a semicircle is 90° , angles in the same segment are equal, opposite angles in a cyclic quadrilateral sum to 180° , tangent at any point on a circle is perpendicular to the radius at that point, tangents from an external point are equal in length, the perpendicular from the centre to a chord bisects the chord, alternate segment theorem.

G11

Basic foundation content	Additional foundation content	Higher content only
solve geometrical problems on coordinate axes		

G12

Basic foundation content	Additional foundation content	Higher content only
identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres		

G13

Basic foundation content	Additional foundation content	Higher content only
interpret plans and elevations of 3D shapes	construct and interpret plans and elevations of 3D shapes	

3.4.2 Mensuration and calculation**G14**

Basic foundation content	Additional foundation content	Higher content only
use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money etc.)		

G15

Basic foundation content	Additional foundation content	Higher content only
measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings		

Notes: including the eight compass point bearings and three-figure bearings.

G16

Basic foundation content	Additional foundation content	Higher content only
know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)		

G17

Basic foundation content	Additional foundation content	Higher content only
know the formulae: circumference of a circle = $2\pi r = \pi d$ area of a circle = πr^2 calculate perimeters of 2D shapes, including circles areas of circles and composite shapes	surface area and volume of spheres, pyramids, cones and composite solids	

Notes: including frustums.

Solutions in terms of π may be asked for. See also [N8](#), [G18](#)

G18

Basic foundation content	Additional foundation content	Higher content only
	calculate arc lengths, angles and areas of sectors of circles	

Notes: see also [N8](#), [G17](#)

G19

Basic foundation content	Additional foundation content	Higher content only
	apply the concepts of congruence and similarity, including the relationships between lengths in similar figures	including the relationships between lengths, areas and volumes in similar figures

Notes: see also [R12](#)

G20

Basic foundation content	Additional foundation content	Higher content only
	know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$ and the trigonometric ratios, $\sin\theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$, $\cos\theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$ and $\tan\theta = \frac{\textit{opposite}}{\textit{adjacent}}$	
	apply them to find angles and lengths in right-angled triangles in two dimensional figures	apply them to find angles and lengths in right-angled triangles and, where possible, general triangles in two and three dimensional figures

Notes: see also [R12](#)

G21

Basic foundation content	Additional foundation content	Higher content only
	know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0^\circ, 30^\circ,$ $45^\circ, 60^\circ$ and 90° know the exact value of $\tan\theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$	

Notes: see also [A12](#)

G22

Basic foundation content	Additional foundation content	Higher content only
		know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$ to find unknown lengths and angles

G23

Basic foundation content	Additional foundation content	Higher content only
		know and apply $\text{Area} = \frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle

3.4.3 Vectors

G24

Basic foundation content	Additional foundation content	Higher content only
describe translations as 2D vectors		

Notes: see also [G8](#)

G25

Basic foundation content	Additional foundation content	Higher content only
	apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors	use vectors to construct geometric arguments and proofs

3.5 Probability

P1

Basic foundation content	Additional foundation content	Higher content only
record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees		

Notes: probabilities should be written as fractions, decimals or percentages.

P2

Basic foundation content	Additional foundation content	Higher content only
apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments		

P3

Basic foundation content	Additional foundation content	Higher content only
relate relative expected frequencies to theoretical probability, using appropriate language and the 0 to 1 probability scale		

P4

Basic foundation content	Additional foundation content	Higher content only
<p>apply the property that the probabilities of an exhaustive set of outcomes sum to 1</p> <p>apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to 1</p>		

P5

Basic foundation content	Additional foundation content	Higher content only
	understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size	

P6

Basic foundation content	Additional foundation content	Higher content only
enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams	including using tree diagrams	

P7

Basic foundation content	Additional foundation content	Higher content only
construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities		

P8

Basic foundation content	Additional foundation content	Higher content only
	calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions	

Notes: including knowing when to add and when to multiply two or more probabilities.

P9

Basic foundation content	Additional foundation content	Higher content only
		calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams

3.6 Statistics

S1

Basic foundation content	Additional foundation content	Higher content only
	infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling	

S2

Basic foundation content	Additional foundation content	Higher content only
interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, and know their appropriate use	including tables and line graphs for time series data	

Notes: including choosing suitable statistical diagrams.

S3

Basic foundation content	Additional foundation content	Higher content only
		construct and interpret diagrams for grouped discrete data and continuous data, ie histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use

S4

Basic foundation content	Additional foundation content	Higher content only
interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:		
<ul style="list-style-type: none"> appropriate graphical representation involving discrete, continuous and grouped data 		<ul style="list-style-type: none"> including box plots
<ul style="list-style-type: none"> appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers) 		<ul style="list-style-type: none"> including quartiles and inter-quartile range

Notes: students should know and understand the terms: primary data, secondary data, discrete data and continuous data.

S5

Basic foundation content	Additional foundation content	Higher content only
apply statistics to describe a population		

S6

Basic foundation content	Additional foundation content	Higher content only
use and interpret scatter graphs of bivariate data		
recognise correlation	know that it does not indicate causation draw estimated lines of best fit make predictions interpolate and extrapolate apparent trends whilst knowing the dangers of so doing	

Notes: students should know and understand the terms: positive correlation, negative correlation, no correlation, weak correlation and strong correlation.